NEW HORIZONS IN MATHEMATICS AS A PHILOSOPHICAL CONDITION: AN INTERVIEW WITH ALAIN BADIOU
Alain Badiou with Tzuchien Tho

Translated with an Introduction by Tzuchien Tho[1]

INTRODUCTION

The following interview takes its origin from the examination of central themes in *Being and Event* after the publication, in 2006, of its sequel, *Logiques Des Mondes*, subtitled "*Tome Deux*." While it seems that this sequel was conceived, on one hand, as an answer to critics and, on the other, as an expansion of his reading of ontology in *Being and Event*, much interest has gathered around the new logical and mathematical tools employed to make such an extension. The "new" system, as it were, takes up category theory instead of sets and a move towards algebraic formalisms. The new approach however also raises theoretical questions about the changes in the status of mathematics and logic in his work. Despite the clarity of his famous maxim that "*mathematics is ontology*," the status of logic, for many of his readers, remains rather mysterious. In *Briefings on Existence*, a text published between the two texts, Badiou makes clear that he sees mathematics and logic as stratified realms, what he calls mathematics pertain to being-as-such and logic pertains to "consistent discourse"[2]. What does this consistency mean and how could mathematics be distinguished from this? This move to "stratify" remained still quite far off from explaining the difference. In *Logiques Des Mondes*, we see finally that while mathematics pertain to being-as-such and logic, by contrast, pertains to "existence" or beings. These precisions, given in the recent publication, provide a new avenue for drawing out the consequences commenced in the first text. It seems that while Badiou answered pressing questions in the expansion of his theoretical project, new and surprising horizons have opened up.

As it appears here, the dialogue with Badiou was an attempt to investigate the new developments by a re-examination of the key points of commencement in *Being and Event*. It revolves around three themes: the status of axiomatics, the relation between difference and negation and Badiou's theory of conditions. Here, I will attempt to unfold slightly the context of the questions, leaving the more technical material for the endnotes.

1.   In *Logiques Des Mondes*, ontology is distinguished from onto-*logy*. As it stands, ontology pertains to being-as-such, and onto-*logy* pertains to existences, and as such, the science of being-as-such would be to mathematics as the science of beings to what Badiou calls "logic." But while onto-*logy* is not ontology, ontology is a kind of being. Hence for ontology or mathematics, there has been reserved a special place in Badiou's theory of beings. Ontology has a logical status. Indeed, this goes a long

way for the attempt to comprehend the distinction between mathematics and logic. Yet, at the same time, this distinction resituates the question in an interesting manner. It is clear, for example, in classical set theory (pick your favorite) that much of what is formalized is done so by at least a modicum of logical references: "and," "if," "not," etc.. Is it plausible that set theory can be distinguished from logic and not thereby dependent on it? If ontology is one among the beings examined by onto-*logy*, does it not mean then that ontology occurs within a context of logic?

2. Much of the project in *Being and Event* requires a particular logical notion of negation and a theory of difference. In *Logiques Des Mondes*, the theory of difference receives much attention insofar as the examination of "appearance" of which Badiou hopes to give an account requires the notion of "intensity." It is no longer simply a question of "yes" or "no," but, in many cases, one of shades and states relative between an appearance and another. The task then in making sense of the consistency between the *two tomes* depends on how one should understand the absolute bi-valence of ontology and its relation with the more diffuse onto-*logy*. One should also not forget that ontology is a being and as such occupies a region in this logical space of beings. How then do we grasp a clear relation between the status of negation and the theory of difference?

3. One of the most interesting themes that traverses Badiou's work is the theory of conditions. Philosophy, Badiou claims, does not produce truths, but rather seizes them from other domains of subjectivity. He remains firm that there are four such domains: science, art, politics and love. This has led Badiou to postulate an interesting reading of the "historicity" of these fields, most notably on the development of "infinity" as it arises in science. "Infinity," a term with various senses, is for Badiou, the core of his tracing of the historicity that he aims to deliver in *Being and Event*, a chronicle of conceptual destiny; as he calls it.[3] There is not a consensus however, at least not in the mathematical community, on the status of this concept. In Cantor's day, the charge was led by Leopold Kronecker who took infinity as something that is little more than a theological remnant parasitic on the advent of modern mathematics. This is not a concern easily brushed off. If infinity is actually a philosophical invention extrapolated from mathematical work, has Badiou simply been dealing with a simulacrum of mathematics and not, as he thought, with the historicity of mathematical subjectivity? In the context of new developments in mathematics itself, how does Badiou see this theme of historicity in his new work?

With these concerns, the interview was aimed at getting a sense of Badiou's present reflections on the new field of investigations opened up by his new work as well as how he sees their continuity with his previous texts. In the end, Badiou does not leave us anything resembling a closed doctrine but rather a charge to investigate these new frontiers.

INTERVIEW

Tzuchien Tho The principle of non-contradiction, a logical principle, is often thought of as a fundamental rule of thinking, or rationality itself. As such it is one of the main examples of an "axiom."[4] In your work, however you begin with mathematical axioms. Are there axioms that come before other axioms? That is, in order to commence thinking, do the mathematical axioms depend upon a prior logical level? The problem for me is how to understand the basic production of set theory in your work with respect to ontology. On the level of axiomatics, I wonder if there is not something more basic there. By pursuing the axiomatic system, do we not remain within a logical framework? Given the plurality of logical systems, classical and non-classical, is there an ontological decision regarding the type of logic employed?

Alain Badiou Does your question relate to the fact that the axioms of set theory are formulated inside a logical framework and hence that would be the real beginning of ontology? I think that the decisions of ontological thought as decisions independent from the context of logic because they are decisions that directly concern the question of multiplicity. Thus that which is thought as pure multiplicity is thought independently of the context of logic. Axiomatics would then be a formal organization of the decision that comes after its realization. The ontological decision, properly said, concerns a multiplicity without the transcendence of the one and then, following that, axiomatics is the mathematical realization of the proper formalization of this decision. Naturally, with axiomatics, one will also have logical choice but the choice would be on a logical world. The world in which one is installed depends on the ontological choice and not the contrary. A precise or technical example, here, is the 'axiom of choice'. In reality, the axiom of choice is an axiom concerning infinity, because there is no problem for the axiom of choice in the finite.[5] In reality the form of the axiom deals with the infinite. This consists in saying that given an infinite multiple of an infinite or finite multiplicity, one can find or seize a multiple composed by an element from each of these multiplicities. Thus, one can decide whether or not [to choose the element in the new set]. I think the fact that the great majority of mathematicians work with the axiom of choice is not simply a pragmatic question. With the axiom of choice, one gets more results but it is also an ontological question. That is to say that I think one should always go in the direction where the multiple is as diverse as possible. There is no reason to restrain or diminish the power of the infinite. Thus if one does not want to diminish the power of the infinite, one should rather accept the axiom of choice. There, one is still in the decision of thought which is not directly joined to the question of logic. Simply put, one finds in this example that if one accepts the axiom of choice, then one's logic would be classical.[6] Here we have a case where accepting the axiom of choice means admitting that one's logical world is classical. One encounters constraints on one's logic. It is not the principle of non-contradiction that is at stake here but the principle of excluded middle, yet one would be obligated to hold the principle of excluded middle not because one wants to admit the principle of excluded middle itself but because if one does not admit it, then one would not be able to have the axiom of choice. And thus I think there is a retroactive effect of the ontological decision in the context of logic. On this point of view, I think the ontological decisions have logical effects that function closer to this, instead of beginning with a logical context and following an ontological decision in the anterior.

I am thinking of another typical example. If you say being is pure multiplicity you evidently have a certain theory of difference. If two sets are different, an element of the one is not an element of the other. Ultimately, this also brings a classical logic to bear because you would see that either one element is in the set or an element is not in the set. This is what founds a doctrine of difference which in turn admits the principle of the excluded middle. Thus the classical character of the ontology of the multiple is constrained by this ontology itself. It is not something developed anterior to a choice of a classical logic, but that which is developed because of the ontology itself. And if the orientation of the principle of choice in mathematics is related to classical logic and not an intuitionistic one, it is not because of logic but rather due to specifically ontological reasons. It is the decision on the ontology, on the pure multiple, on the axiom of choice, on the infinite. After this moment, it will be logically constrained but as something after the fact.

That is the first part of our question. After all, what must not be overlooked is that there are non-classical worlds, but it is the worlds themselves that are not classical. The logic of being *qua* being remains classical. There is no contradiction between the fact that the logic of pure being remains classical and the logic of the localization or the logic of appearing can be non-classical. If one can think in-itself a logical horizon in the classical, there will also be, in excess, a transcendental that would very well be non-classical. We distinguish between the theory of multiplicity as such and a theory of multiplicity localized in a transcendental of a world. The intuitionist is attached to concrete experience because they work within the context of one world. But that would be an intuitionistic world. Instead, pure being or pure multiplicity is necessarily in a classical world.

I think that one can develop a general theory relative to what we have explained concerning the decisions in pure ontology (that constrains us within a classical world) but it may be that any empirical world would be perfectly ruled as a non-classical world. That is not contradictory. As we read in *Being and Event*, ontology itself is a situation, ontology is a world which is classical precisely for the reason that it holds as a world.

TT For this reason, I reexamined the beginning chapters of *Being and Event* and at the most basic level of articulating the nature of the inconsistent multiple, on the one hand, and consistent multiplicity, on the other hand, and in relation to Cantor also, you use a proof with the logical principle "*ex falso quodlibet*" to demonstrate the universal inclusion of the void.[7] There I recognize something that requires a strong notion of negation. This suggests a way of accessing the question of the inconsistent multiple itself not simply on the level of presentation or demonstration within ontology but as that which allows for such a kind of presentation, perhaps a kind of negativity, a kind of negation or inconsistence, from the outside, that governs or "disciplines" it.

AB On the question of the strength of negation, I think there is no difference between the question of negation and the question of the different frameworks of logic. Because we know that there are fundamentally three different types of logic, the paraconsistent logic, intuitionistic logic and classical logic and these express exactly three types of negation.[8] Among my projects, I am planning to write a short essay concerning negations, specifically concerning the three types of negation and the philosophical tradition. What is exactly the dialectical negation, or the negation in Plato and other sorts of negations? So, I want to organize something like a history of negation across the question of difference between logic and ontology.

I agree with you that the use of "*ex falso quodlibet*" or apagoic reason, in the proof by absurdity, implies a very strong notion of negation but it is in fact a realization at one point, the point of negation, of the strength of classical logic itself. When we not only have the principle of non-contradiction but also the principle of excluded middle, we have by necessity a very strong concept of negation. But I think that it is also a consequence of the ontological decision. Given a multiplicity, the strict determination of multiplicity will be at the level of elements. You have by necessity "to be in" and "not to be in" the multiple, there is a complete gap, a complete separation and this separation is intelligible only with a strong concept of negation, because it is either "yes" or "absolutely no" and no other possibilities. So I agree with you when you say that it is not only in the demonstration, in the discourse of exposition of ontology, that there is that sort of use of that negation, but in fact in the very sense of the ontological decision itself.

TT Perhaps then there is a prior decision on negation, not concerning the multiple as such, but a kind of principle?

AB I don't completely agree because I believe that the force of negation is a consequence of the essential character of the void. It is a consequence, in reality, of the axiom of extensionality.[9] The fundamentally absolute decision is the regulation of multiplicity with the axiom of extensionality. That is to say, multiples are different if and only if there is an element that is in one that is not there in another. In this case, I say that the absolute decision is not concerning negation, but rather difference, but it is close. Perhaps, as with Hegel, thought begins with difference but ends up depending on negation all the same. Here I think, technically and precisely, the absolutely constitutive decision that leads towards whether a world is classical, where the excluded middle holds or if there is a reason by the absurd, is the axiom of extensionality or what is sayable through the axiom of extensionality.[10] What is this decision? It is that there is always a point which raises the question of difference. I think that there is an absolutely fundamental choice here. Or, if there is a difference between two totalities either there is a point at which one can say that "this" is why it is different [or there is not]. Thus it is finally a decision with respect to the global and the local. This becomes very interesting once you have decided that you can always localize the difference. It is not the difference between black and red for example. It is not founded in the world. If you have decided that the difference is always localizable then by consequence you will have a strong concept of negation. In a certain sense, the classical dialectic of Hegel, does not take this decision. On the contrary, he takes this difference to be a global category and very close to identity and thus one has a negation which would not be strong but rather a negation which will work or transform but not a negation which strictly excludes. It is simply for this that your question is justified. Because when we have a strong negation, it takes us toward a void that is everywhere. Thus the void is the realization of a strong negation in the form of the existence of a set which has no elements, with no local identity because it has no elements. There exists one and only one such set. We would see this but only as a consequence. The original idea from the Greeks, which in my opinion created mathematics, is that one can think the pure multiple as something which always has a localization of difference. After that we have evidently a classical logic and a strong negation.

TT Perhaps this is reflected in a question of translation. The term "consistent" in English has a sense of something free of contradiction—the inconsistent set is the set that we talk about in "*ex falso quodlibet*". As I understand it, in French, the sense of the term "*consistance*" does not have a strong sense of contradiction, but rather something which is not "formed" or not "collected."

AB Yes. I think the meaning of consistent and inconsistent is not exactly the same, I would not go as far as to say that that is the case generally for the French language, but at least in my usage, it is not the same thing and your explanation is perfectly correct.

TT On the basis of this difference, I wonder whether the inconsistent multiple would not be split between two senses of the inconsistent, in your sense and in the sense of the contradictory. The reason why I think this is important is because relative to the position of the void in the two systems that you have developed in *Being and Event* and *Logiques des Mondes*, there is a sort of stabilization point which employs a strong sense of negation. In pure ontology there is a stabilization point in the void and with the question of existence, it is negation that establishes a minimum in relation to the "inexistent", so in a way, in the most specific local point, there is also a global effect.[11]

AB I perfectly understand your question which finally begins to force me to confess my primordial decision concerning negation or my taking negation as a fundamental principle. (*Laughs*) I exaggerate, but there again, the void, as you say, the void of ontology or the inexistence in appearance or in the world, in these two cases there is the negative or a space where there is something of importance given to the negative. It is true, I would say yes, but I think that it is a consequence, and not a principle. The discussion is uniquely there, whether it is to be a principle or a consequence. The reason why the void is important is because it is the moment where there is a null point of identity. It is important solely because the decision taken concerning difference cannot be applied to the void because if difference is always made, in this case, element by element, what do we do with difference of the void where there are no elements? Then there will be the void as the moment indifferent of being. Thus it will be unique because one cannot differentiate a different sort of void. Thus the primordial character

of the void is a consequence of the primordial character of difference. But with the idea of existence, we have the same thing because existence is nothing but a network of differences. The inexistent is something among these differences, but as something that is never expressed in the positive. There, it is also that which is not identifiable by the differences in the interior of appearance with the others. In both cases, there is a first decision concerning difference: in ontology, the difference is always local, and in appearance, where it is in contrast always global because an object is a network of difference. But inexistence is that which, in this network of relations, permits the naming of identity, but which is thus absent. It is present as an absence. In both cases, however, while it is true that these factors bring the force of negation to bear, it arrives as a consequence and not as a principle.

TT Maybe we can add something to this discussion from the question of the process of fidelity. In the above, we speak of ontology in general, but with fidelity we speak of a subjective principle of enquiry, the designation of generic sets. In the second part of *Being and Event* there is a very strong principle of negation in deductive fidelity. My question is whether there might be another way of getting at this subjective principle.

AB Yes. In *Being and Event*, I studied exclusively the ontological destiny of fidelity. What interested me in *Being and Event* was demonstrating that there exists a type of multiplicity which corresponds effectively to the production of fidelity which is the generic multiplicity, a generic subset. Thus it is an intra-ontological demonstration concerning fidelity and that is to say, effectively, that there is a real possibility to think the result of the process of fidelity as a multiple and we would not have to search for something outside ontology itself. It is not something like another type of being, so the truth is not transcendent. Finally then, truth is inside the situation. Truth is a multiplicity as I have said, as well as the demonstration, the deduction of all that concerns the concept of generic multiplicity. With respect to the ontology, we have a deductive framework.[12] In *Logiques des Mondes*, it is different because I examined the possibility of a type of fidelity outside the strictly ontological framework. Sometimes I say that it is as if, in *Being and Event*, I examined the skeleton of the beast, where the beast is truth, and in *Logiques des Mondes*, I examined something which is not at all the skeleton but which is the flesh and the concrete elements which compose the different types of truth. So it is not the same level, and there is absolutely a possibility of a fidelity which is not strictly deductive if the world is not a classical one. I say this with an additional remark on the question of points.[13] Because in all processes of truth we have a series of points upon which we return to the classical logical framework because the point -in a world which is non-classical -is nonetheless classical insofar as, in each, it is something to which there is a "yes" or "no." There are two possibilities and only two. So we have a mixture, finally, of something which is non-classical, maybe something which is the concrete becoming of a truth in a non-classical world and we have there all the elements of the truth, of the body of the truth, which is under a law that is not classical. Here, we have a process of fidelity which is non-deductive and, in addition, a mixture of something else which is a process made point by point. Yet, when we have to confront the point, the question is finally a "yes" or "no," a classical question.

TT So, from the perspective of evaluation, when we have a world, a non-classical world, where the points relate back to a classical model, we have a relation of reflection. Is there perhaps a relation between the question of logic in *Logiques des Mondes* and David Lewis' modal logic.

AB There is something like that in the structure of the point because precisely it occurs in a mixture that is not itself reducible to the strictly deductive construction. It is not exactly true or false, but something much more complex in the relationship between situations which resemble shades, where we have something between true and false and all those non-classical constructions, maybe even sometimes with

infinite possibilities. It is a relation between this and the most rigorous or strict classical universe, the little matrix with only minimum and maximum, 0 and 1. It is a relationship between the two which is finally the true element of the truth process. I think that we can find this in concrete experiences, in fact, because there is something like a becoming of a strict choice, or something like a concentration on one point of all the complex processes which come before that point.

TT So one can say that in that relation [between worlds and points], there is a kind of filter, perhaps, of all the points that correspond to the bi-value, what precedes it, and perhaps then there might also be another filtering point, through poetry maybe, which perhaps does not require us to pose the bi-value.

AB There is something like that. In the most complex worlds, where there might be something like the necessity of poetic language, we find subtleties, shades, and the construction of a truth in a sort of infinite language across infinite situations. In the end however, there will always be a point or moment where we have to provide the proof of all of that, all those complex constructions that sustain or traverses the difficulty or exigencies of a commitment to a point. And I have provided some of these proofs in the world of art, not only in poetry but music and such. There are always some moments in which we have to concentrate all of the process and find some decisions which retroactively validate the complexity of the whole process. We can perfectly examine the construction of the work of art in music, to use a clear example, some moments that are not like the rest of the composition, and we will find this point inside the construction itself. I think that, in general, the strictly classical world is a mathematical world, the ontological world as such. All generic multiples that are not in the mathematical framework are generally not in a classical logical framework at all. The relationship between the poetic process and the construction of a generic truth is finally something like the relationship, the real relation, between life and mathematics. But of course this is… a metaphor. (*Laughs*)

TT That is a nice way to think the next question, the theme of infinity. Infinity is a feature that is found in almost every aspect of your work. Perhaps we can say, in a reading of your work that, aside from the problem of infinity thought through ontology, in natural and historical situations, there is also an infinity that unfolds according to the subjective principle. Zachary Fraser, who is translating *Le Concept de Modèle*, has recently raised some questions on infinity thought through the generic procedure and infinity thought through intuitionistic choice sequences as Brouwer proposed.[14] In relation to the article of Saul Kripke, "Semantical Analysis of Intuitionistic Logic", he shows that there is an isomorphic relationship through the temporality of subjectivity.[15] One conclusion that one might draw here is that generic infinity would seem to be much smaller than what you often speak of, the transfinite. This would reduce the infinite to something like the "indefinite." Does this pose any problems? Further, I was wondering if something like the "unnamable," which is something of a point within the truth procedure that delimits the infinity of the process, is in a way to respond to Fraser's criticisms.

AB I am not entirely satisfied with the description of the truth procedure as such in *Being and Event*, there is something too elementary there. I have read the paper by Fraser and thought about your [earlier] questions concerning the possibility of thinking about infinity, continuity and temporality in a different manner or, in this case, to adopt something like Brower's choice sequences in order to think continuity as such, rather than as a set, these open a field for new questions.[16] I accept that. My goal in *Being and Event* was not to examine the types of infinity of the truth process but much rather to clarify the possibility of the construction of a generic subset with the successions of moments, which are named "enquiries" and I understand that there is something close to the construction of something like infinity in the intuitionistic framework. There are successive choices and so on. Maybe it is not important for me to know whether the succession of the truth process is classical or intuitionistic. I can accept that ultimately the intuitionistic process is much more like the process [of enquiries] itself because it is not an ontological question. It is the question of the subject and the question of the subject is not exactly an ontological question. I can accept that a new and very interesting field of discussion concerning the relationship between, on the one hand, an ontological conception of infinity in a Cantorian style, and on the other hand, all the concepts of intuitionistic logic, the sequences of choices and the semantic framework of Kripke is possible. The fact that there is a reconstruction in that sort of context of the technique of forcing

is very interesting. So I have no dogmatic position on this question here. I completely accept that at the level of the logic of worlds, it may very well be that the construction of a truth process occurs in an intuitionistic context because it takes place distinct from ontology and ontological constructions. So that sort of question is for me a real question and not a confrontation. We have to work, you, me and others. This sort of question is very complex and takes on new mathematical methods and contexts. I have been very interested in Kripke semantics for a long time but we can perfectly see that in *Being and Event* itself the truth process is not formally temporal. While we have successive inquiries, I do not at all place that in a temporal series, but it is a possibility. I have always said that there is an event and after an event, a truth process. It is a construction of a new time, but I have not done much work to make this concept more precise. But the general idea is really that there is finally something temporal in the very process of truth, not in time itself but the generation of a new time.

TT Do we turn, with this, to a question of method, the relativism of method?

AB I am absolutely open to new methods. My complete proposition including what is developed in *Logiques des Mondes* is that there is no real fundamental choice of an absolutely singular mathematical framework. I use a theory of categories as well as a theory of sets. And so, if something concerns pure multiplicity, then I remain [with the theory of sets]. But there might be another means to formalize all that. Different ways with different conclusions, a new series of time or something like that, something might be different concerning the very concept of the subject. It is good news for me if it follows from a development proper to my intuitions. It is not a closed project where I have to defend the details of the mathematics. A much more important task for me is the affirmation that the different means in the mathematical field today may have real philosophical consequences. It would not be difficult for me to begin with an agreement with something like that but maybe after that point we can begin to difficulties or discussions concerning the consequences in that context. But I am convinced however that it is always after this that we will have very interesting discussions, and not a strict incompatibility. There is not at all an incompatibility between different logical possibilities because they concern different worlds and different worlds do exist—in fact.

TT So your new framework in *Logiques des Mondes* explains the specificity of *Being and Event* as a world among others and leaves ultimately a more expansive view?

AB We have to read *Being and Event*, or I have to read *Being and Event* within the opening of something like a new articulation between ontology, logic, mathematics, event, subject and so on -a new articulation of all of that. As often in philosophy, the style is more important than the details. And it is very important to admit that there is a fundamental function of the formalism in mathematics and the formalism of logic in philosophy itself, not only as an object for philosophy. We have to reconstruct the Platonic conviction that mathematics is something with an intimate relationship with philosophical concepts themselves. If mathematics is ontology, there is a great history of ontology and at each stage we have then naturally a new philosophical conception. I say this concerning Cantor but I also mean that Cantor is not the end of mathematics.

TT What you say about Cantor is interesting. When Leopold Kronecker reviewed Cantor's work, he said that, "I don't know what predominates in Cantor's theory—philosophy or theology—but I am sure that there is no mathematics there." On the question of infinity, there is a question of whether it is a mathematical concept or a philosophical one.

AB I understand perfectly why Kronecker said that and it is in the idea of Cantor himself. Because, you

know, Cantor was very tormented by the idea that with the actual infinite there is something that is incompatible with the Christian religion. Cantor was mad. (*Laughs*)

TT Maybe there is something similar with Galileo here.

AB Absolutely. I agree with Kronecker on one point. Cantor's theory is really of such great philosophical consequences that maybe the philosophical consequences are more important than mathematical ones. Because, you see, set theory is not as important today as part of concrete mathematics. Algebraic topology maybe, but set theory is, maybe, a finished theory. Perhaps like the great concept of categorical algebra, it is not very important as an abstract framework. But the philosophical consequences might be more important. However, I agree with Kronecker on the first part but there is also a false part - there *is* mathematics there (*laughs*). The false part is that there is no mathematics there, but that is not true. The mathematical destiny is probably less important, for the general history of thinking, if you want, than the philosophical consequences.

TT The reason why I raised this concerns the separation you make between philosophy and ontology. In this separation of philosophy and ontology, there is a theory of conditions and so my question concerns whether the theorization of infinity is really a mathematical inscription at all or really a philosophical one. As a matter of conditions, the question is whether the question is not really a question of philosophical conditions themselves. Is infinity finally only a philosophical and not mathematical question?

AB In fact, there is something like a retroaction here because the question of the philosophical consequences of a new mathematical concept is not by itself a mathematical question naturally. That Cantor's new mathematical construction of the concept of the transfinite is something with philosophical consequences is, finally, itself a philosophical decision. While philosophy is conditioned by new mathematical creations, the choice to assume this condition and its philosophical effects are found within the field of mathematics itself. In fact, there is a retroaction such that the very concept of the infinite becomes a philosophical question. When you decide that Cantor's discovery is a new condition for the philosophical question of the infinite, you have a reading of mathematics which is not a reading of mathematics by mathematicians. It is something else. Finally, to become a new condition of philosophy is something that happens to mathematics and to politics… or to art. It is something that happens to philosophy first, but after that you have something which philosophically happens to the condition itself. So it is not entirely an innocent thing to become a condition for philosophy because after that something happens to you. (*laughs*)

TT And to the world?

AB Absolutely.

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Tzuchien Tho is a graduate student of early modern mathematical concepts at the University of Georgia (Athens, USA). He is currently in Paris doing research on mathematics and its impact on contemporary approaches to ontology and writing a dissertation on Leibniz's work on the continuum and infinitesimals.

NOTES

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2. Alain Badiou, *Briefings on Existence*, trans. Norman Madarasz. Albany: State University of New York Press, 2006,159.

3. Alain Badiou, *Being and Event*. trans. Oliver Feltham, London and New York: Continuum, 2005, 281.

4. A good example of the historical importance of the role played by the principle of non-contradiction is Aristotle's discussion in passages from Metaphysics Γ.

5. The axiom of choice is an independent axiom of set theory that states: $\exists x[\forall y(y \in x \rightarrow \sim(y=\emptyset)) \& \forall y \forall z(y \in x \& z \in x(y=z) \rightarrow \sim(\exists w(w \in y \& w \in z))) \rightarrow \exists u \forall y(y \in x) \rightarrow \exists z(z \in u \& z \in y \& \forall w(w \in u \& w \in y \rightarrow w=z))]$ or, "If *a* is a set, all of whose elements are non-empty sets no two of which have any elements in common, then there is a set c which has precisely one element common with each element *a*." Cf. Mary Tiles, *Philosophy of Set Theory*. Mineola: Dover Publications, 2004, 123-134. Without going into the details, an example of the principle behind the axiom would be the following. Suppose one has a number of bags containing different fruits, then it is possible to take exactly one fruit out of each bag to make a different bag of fruits. In the finite cases, this new bag of fruits can intuitively be grasped to exist. It is less intuitive to assert that such a bag exists in the infinite case. The axiom of choice asserts that this new bag or set *exists* in both finite and infinite cases such that if one had an infinite set of bags, such a new bag of fruit is affirmed to exist. In cases involving infinite sets, the problem arises when it is unclear how such a choice from each non-empty set can be made. In other words, in these problematic cases, it is not clear how one could construct or otherwise ascertain such a set without knowing what it is that is being "chosen." Badiou refers here simply to the fact that the axiom of choice is problematic with regard to infinity. In *Being and Event*, Badiou argues in meditation twenty-two that the axiom of choice presents the "form-multiple of intervention" and formalizes the axiom as $(\forall\alpha)(\exists f)[(\forall\beta)[(\beta \in \alpha \& \beta \neq \emptyset) \rightarrow f(\beta) \in \beta]]$, where the axiom affirms the existence of a "function f, such that, if is the given set and if $\beta \in \alpha$ we have $f(\beta) \in \beta$" Badiou employs the notion of a function to explain the significance of this axiom instead of the more elementary presentation above. Cf. Badiou, *Being and Event*, 223-231.

5. While there is a wide proliferation of logics in contemporary scholarship, three basic forms are recognized widely. They concern arguments over two different principles: the principle of non-contradiction $[\sim(a \& \sim a)]$ and the principle of the excluded middle $(\sim\sim a \& a)$. Classical logics are identified by their adherence to both. Intuitionistic logic rejects the principle of excluded middle. Paraconsistent logics reject the principle of non-contradiction. Badiou's recent *Logiques des Mondes* affirms that not all worlds are classical. In particular, in book II section 5, Badiou formally distinguishes between an intuitionistic and a classical world. Cf. Alain Badiou, *Logiques des Mondes*. Paris: Editions du Seuil, 2006, 195-200. For examples of the proliferation of various logical forms and the philosophical stakes behind them see Dale Jacquette (ed.), *A Companion to Philosophical Logic*. Malden, Oxford and Victoria: Blackwell Publishing, 2002.

6. Badiou uses "*ex falso quodlibet*" $(\sim a \rightarrow (a \rightarrow b))$ to demonstrate the universal inclusion of the void set. Cf. Badiou, *Being and Event*, 86.

7. Many theorists of paraconsistent logics attempt to weaken the principle of non-contradiction because they find classical tautologies like "*ex falso quodlibet*" objectionable for philosophical reasons. These arguments pertain to the "strength" and meaning of negation. A variety of camps engaged in this debate distinguish between relevance logic, paraconsistency and dialetheism, developing a number of ways to handle the question of contradiction and negation. Cf. Graham Priest, *In Contradiction: A Study of the Transconsistent*. Oxford and New York: Oxford University Press, 2006.

8. Badiou's own formalization of the axiom of extensionality reads: $(\forall\gamma)[(\gamma \in \alpha) \leftrightarrow (\gamma \in \beta))] \rightarrow (\alpha=\beta)$. As he explains, "It indicates if, for every multiple γ, it is equivalent… to affirm that it belongs to α or to affirm that it belongs to β, then α and β indistinguishable and can be completely substituted for each other." In other words, a set has no identity other than what belongs to it. Badiou, *Being and Event*, 61.

9. In *Being and Event*, Badiou dedicates Meditation Fifteen to Hegel where he argues against the "continuous dialectic." Instead, "Mathematics occurs here as discontinuity within the dialectic." Badiou, *Being and Event*, 169. At this juncture of the interview, Badiou focuses rather on the question of negation in Hegel's dialectic in the wider sense. Badiou seems to echo some of Gilles Deleuze's critique of Hegel's dialectic and representation in *Différence et Répétition*. Cf. Gilles Deleuze, *Différence et Répétition*. Paris : Presses Universitaires de France, 2005, 342-345.

10. Badiou provides a very concise discussion of the logical framework of a "classical" world based on the definition of a transcendental from the interplay of "maximum" and "minimum" in book II, section five of *Logiques des Mondes*.

Cf. Badiou, *Logiques des Mondes*, 199-200.

11. In *Being and Event*, Badiou spends the twenty-fourth meditation spelling out how deduction constitutes an operator of ontological fidelity. Dealing formally with *modus ponens*, reasoning via hypothesis and *reductio ad absurdum*, Badiou holds the three kinds of deduction together as the "equivocal paradigm of fidelity." At this juncture of the interview Badiou clarifies that this paradigm is an approach from an ontological framework rather than a phenomenological one. Cf. Badiou, Being and Event, 240-254. A more in-depth analysis of logical deduction and its central importance to Badiou can be found in his 1969 text *Le Concept de Modèle*. Alain Badiou, *Le Concept de Modèle*. Paris: François Maspero, 1969. English translation forthcoming as *The Concept of Model*, trans. Zachary Fraser, Melbourne: re.press, 2007.

12. In *Logiques des Mondes*, Badiou spends some time developing a theory of points that allows him to formally relate his distinction between "simple becoming" and "real change" with his concept of worlds. Cf. *Logiques des Mondes*, 421-470.

13. Cf. Zachary Fraser, "The Law of the Subject: Alain Badiou, Luitzen Brouwer and the Kripkean Analyses of Forcing and the Heyting Calculus," in *The Praxis of Alain Badiou* (ed) Paul Ashton, A.J. Bartlett & Justin Clemens, Melbourne: re.press, 2006, pp. 23-70. See also, *Cosmos and History*, vol. 2, no.1-2, pp.94-133.

14. Cf. Saul Kripke, "Semantical Analysis of Intuitionistic Logic I", in J.N. Crossley and M.A.E. Dummet (eds.), *Formal Systems and Recursive Functions, Proceedings of the Eight Logic Colloquium, Oxford July, 1969*, Amsterdam: North-Holland Publishing Co., 1965, pp.92-130.

15. Badiou refers here to questions from an earlier informal discussion where I posed questions concerning continuity and discreteness in mathematics. That there is a thematic tendency for Badiou to champion the discrete over the continuous is known but it is particularly interesting to read Badiou's comments here against his 1968 article *La Subversion Infinitésimale*." In the text, he writes, "Qualité, continuité, temporalité et négation: catégories asservissantes des objectifs d'une idéologie. Nombre, discrétion, espace et affirmation ; ou, mieux, Marque, Ponctuation, Blanc et Cause : catégories des processus scientifique." Badiou pronounces continuity and negation as being "servile to ideology" and relates the discrete to "scientific process." Alain Badiou, "La Subversion Infinitésimale," *Cahiers Pour L'Analyse*, no.9, Summer 1968, pp.118-137, 136.